



11 The adjoint

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Introduction

- In this topic, we will
 - Define the adjoint of a linear map
 - Demonstrate that it is a linear map, too
 - Describe what will be covered in the following topics





The adjoint

- Given the linear map $A: \mathcal{U} \rightarrow \mathcal{V}$, then we will define the *adjoint* of the linear map as that map $A^*: \mathcal{V} \rightarrow \mathcal{U}$ such that

$$\langle A\mathbf{u}, \mathbf{v} \rangle = \langle \mathbf{u}, A^* \mathbf{v} \rangle$$

- The existence of the adjoint is an upper-year topic in pure mathematics, hence we will simply assume it exists
 - You may look up the *Riesz representation theorem*





The adjoint is a linear map

Theorem

The adjoint is linear.

Proof:

$$\begin{aligned}\langle \mathbf{u}, A^* (\alpha \mathbf{v}_1 + \beta \mathbf{v}_2) \rangle &= \langle A\mathbf{u}, \alpha \mathbf{v}_1 + \beta \mathbf{v}_2 \rangle \\ &= \alpha \langle A\mathbf{u}, \mathbf{v}_1 \rangle + \beta \langle A\mathbf{u}, \mathbf{v}_2 \rangle \\ &= \alpha \langle \mathbf{u}, A^* \mathbf{v}_1 \rangle + \beta \langle \mathbf{u}, A^* \mathbf{v}_2 \rangle \\ &= \langle \mathbf{u}, \alpha A^* \mathbf{v}_1 + \beta A^* \mathbf{v}_2 \rangle \quad \blacksquare\end{aligned}$$





Upcoming lectures

- In the upcoming lectures, we will:
 - Look at the properties of the adjoint map
 - Determine what the adjoint of a real matrix is
 - Describe what the adjoint of a complex matrix is
 - Reinterpret the inner product
 - Describe self-adjoint and skew-adjoint operators
 - Consider some consequences of such operators
 - Describe normal operators
 - Describe isometric operators
 - Describe linear regression





Summary

- Following this topic, you now
 - Understand the definition of the adjoint of a linear map
 - Know that it is a linear map
 - Are aware of future topics





References

- [1] https://en.wikipedia.org/wiki/Hermitian_adjoint





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas. Mathematical equations are prepared in MathType by Design Science, Inc. Examples may be formulated and checked using Maple by Maplesoft, Inc.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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for more information.





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